PRF with Weak Programming Privacy
and Applications to Private Information Retrieval

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joint work with

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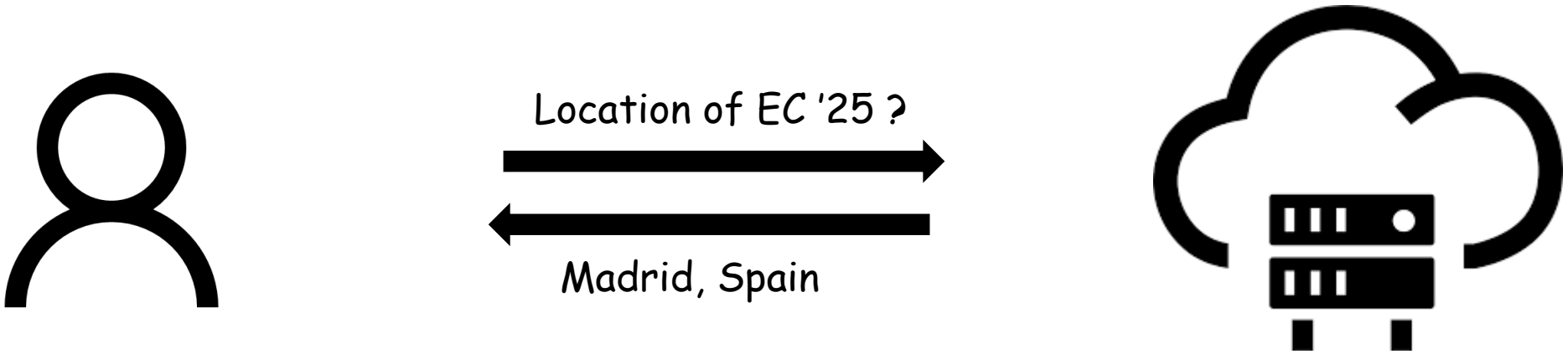
Carnegie Mellon University

Information Retrieval Leaks User Privacy



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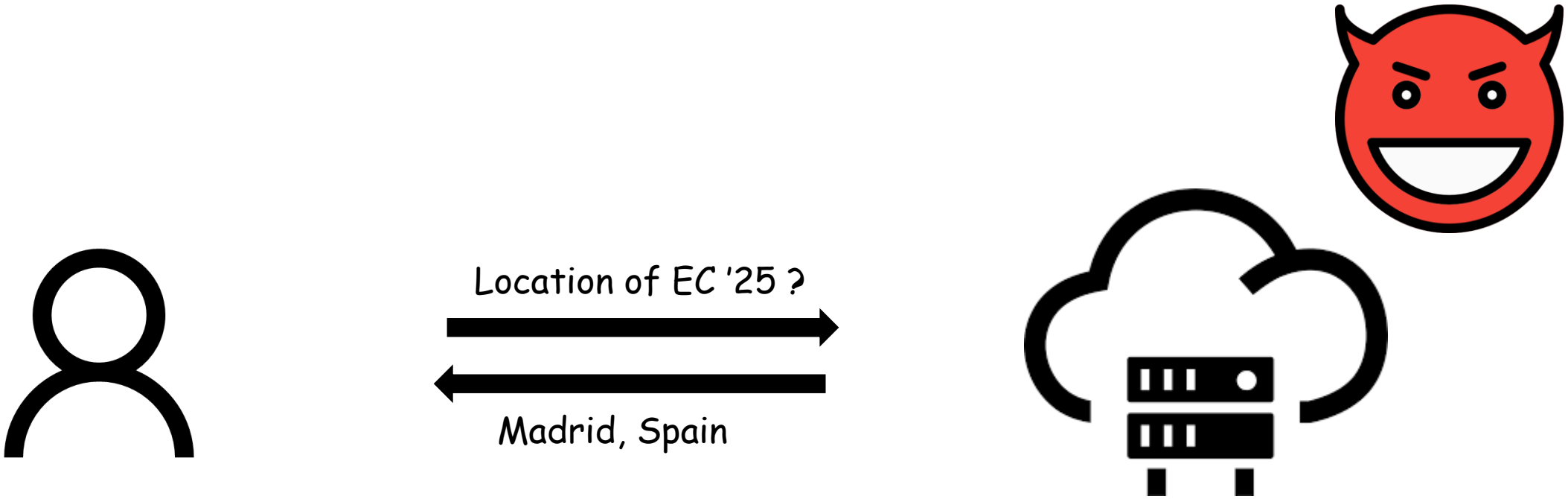

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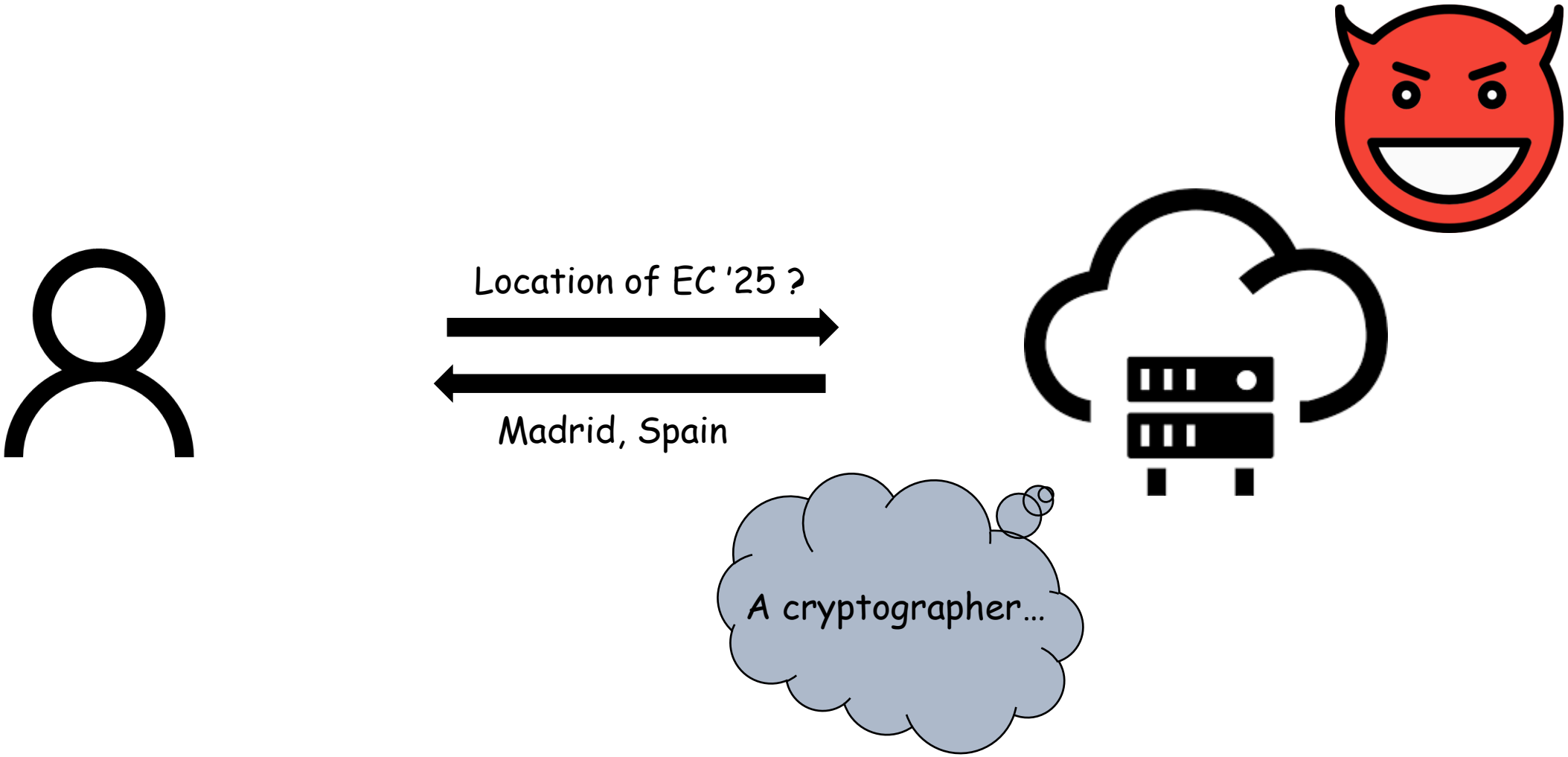
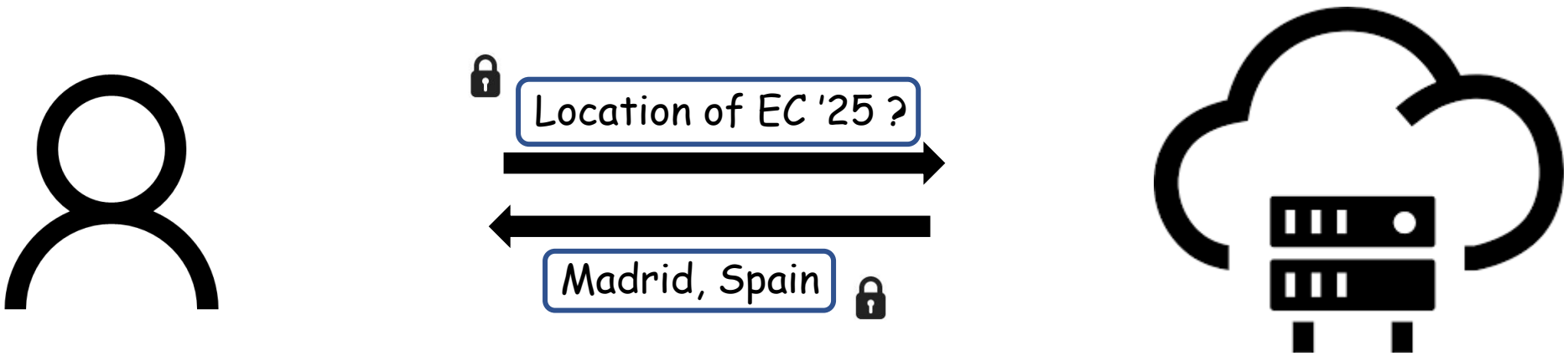


Private Information Retrieval [CGKS95]



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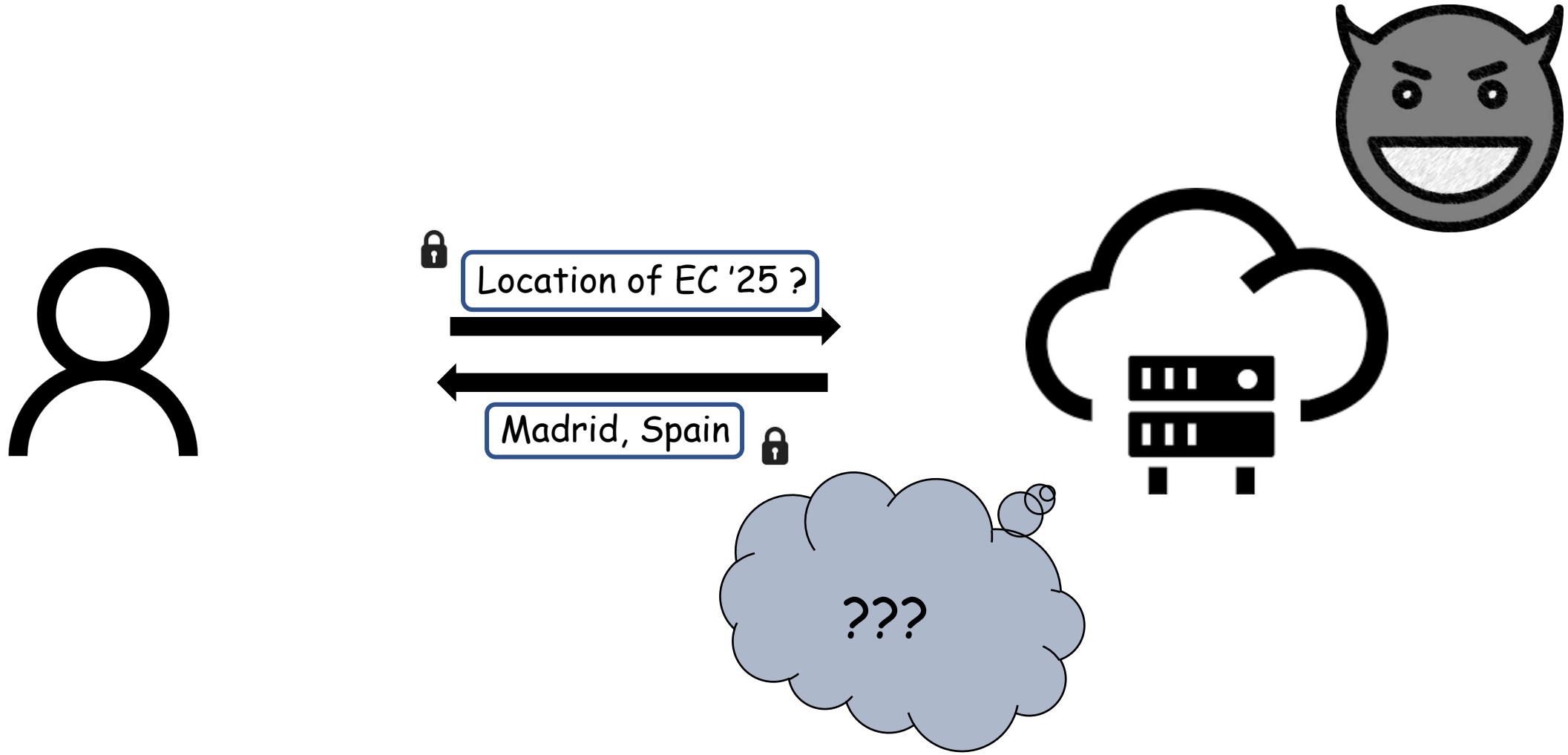
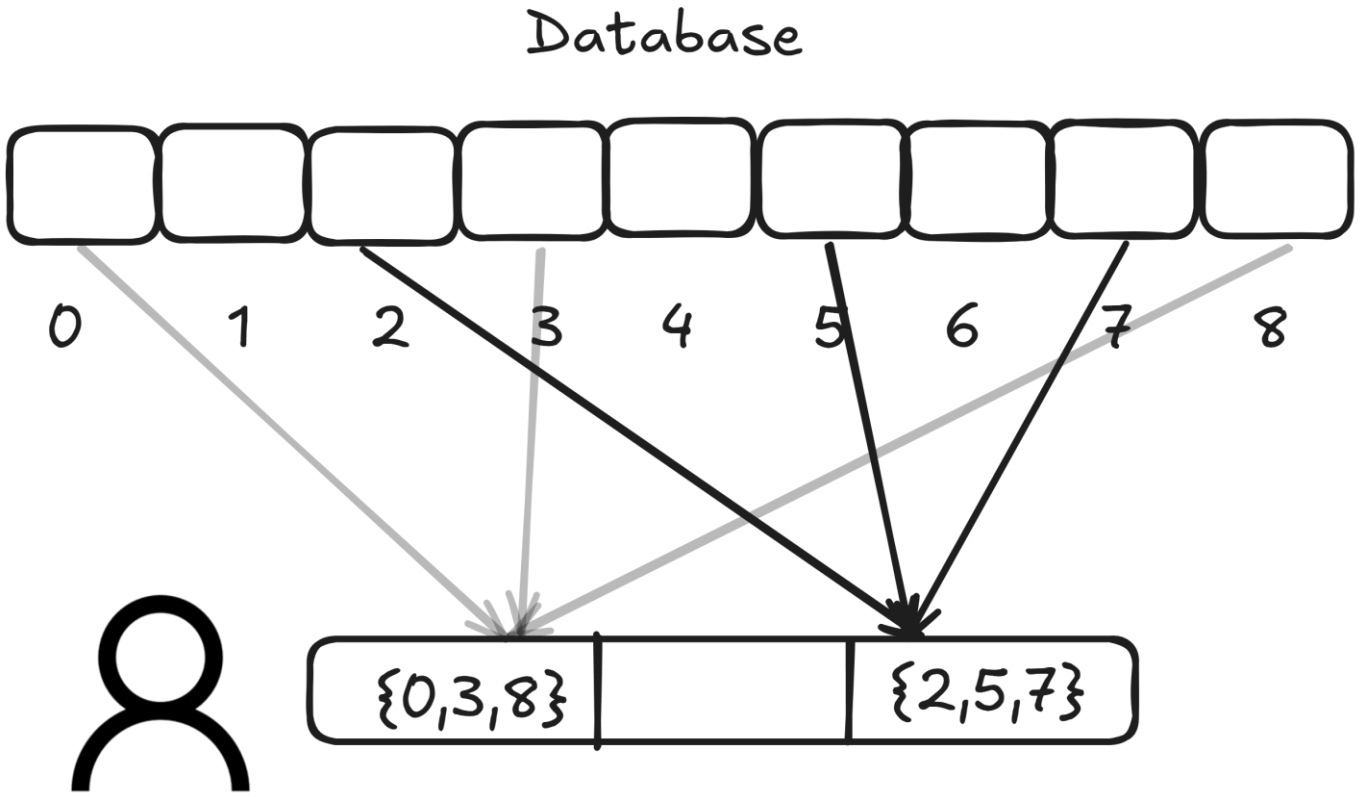


Recently…

- Advance on Preprocessing PIR
  - [BIM00],[CK20],[KC21],[SACM21],[CHK22],[LP22],[LP22b],[HHCM+23],[ZLTS23],[LMW23],[LP23],[MIR23],[ZPSZ24],[HPPY24],[GZS24],[ISW24]

- Many of them follow a common framework

Preprocessing



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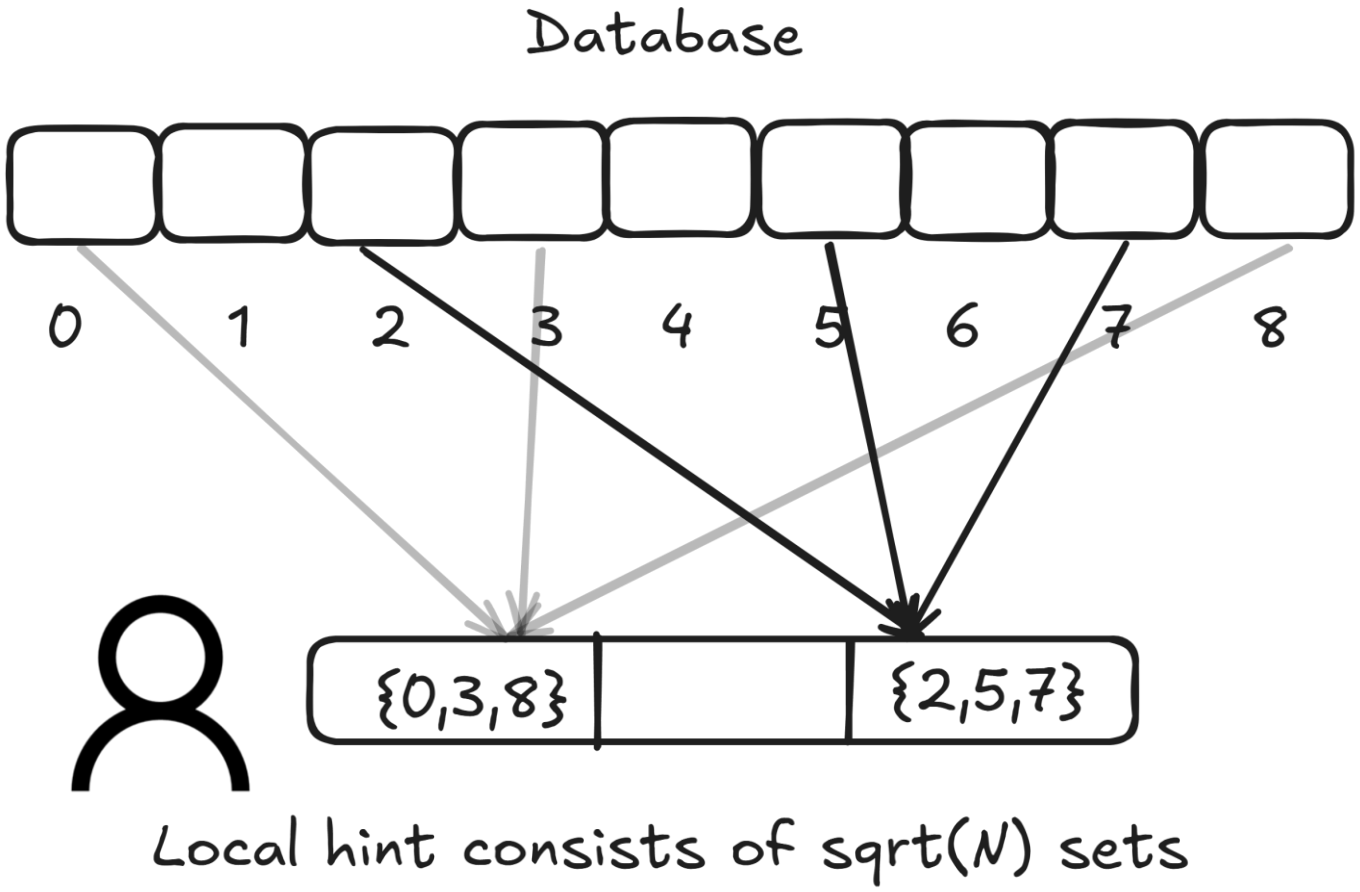

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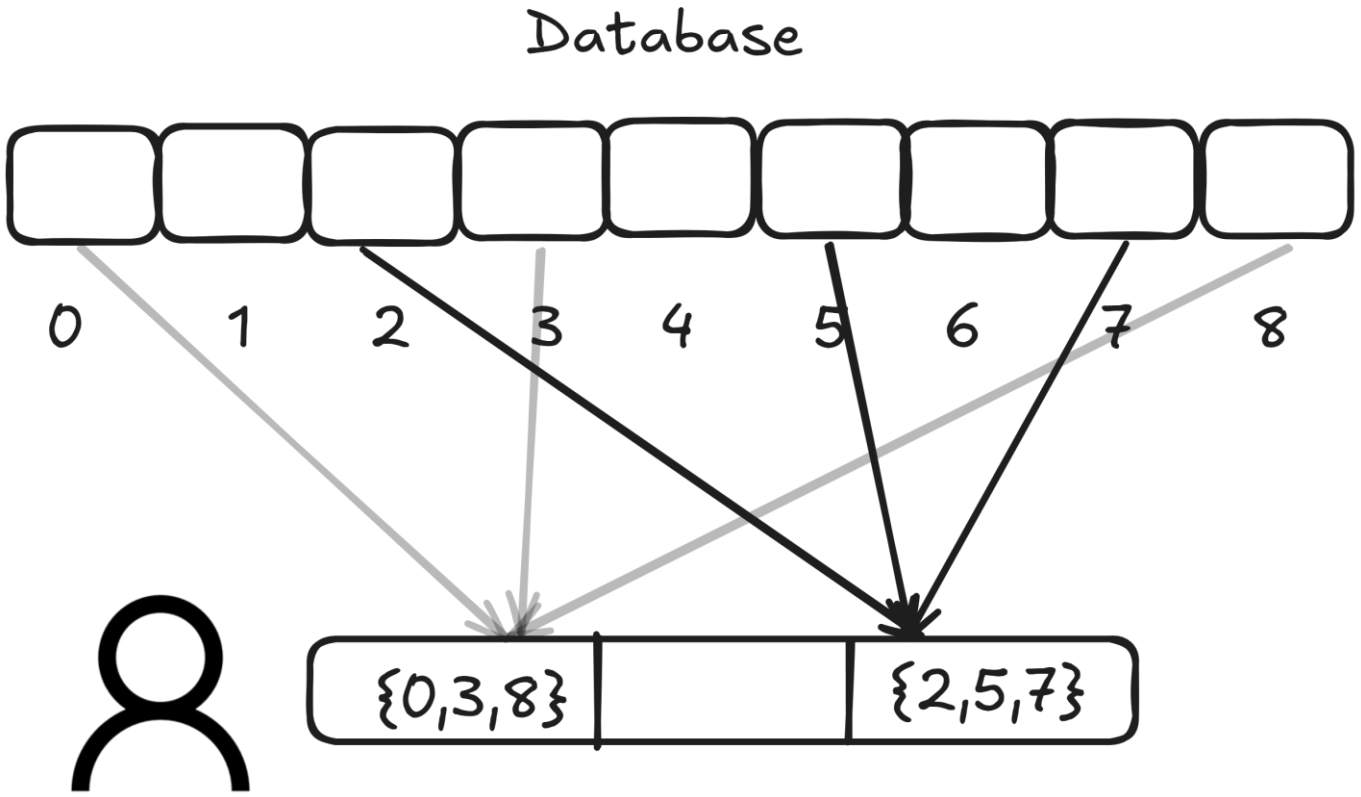

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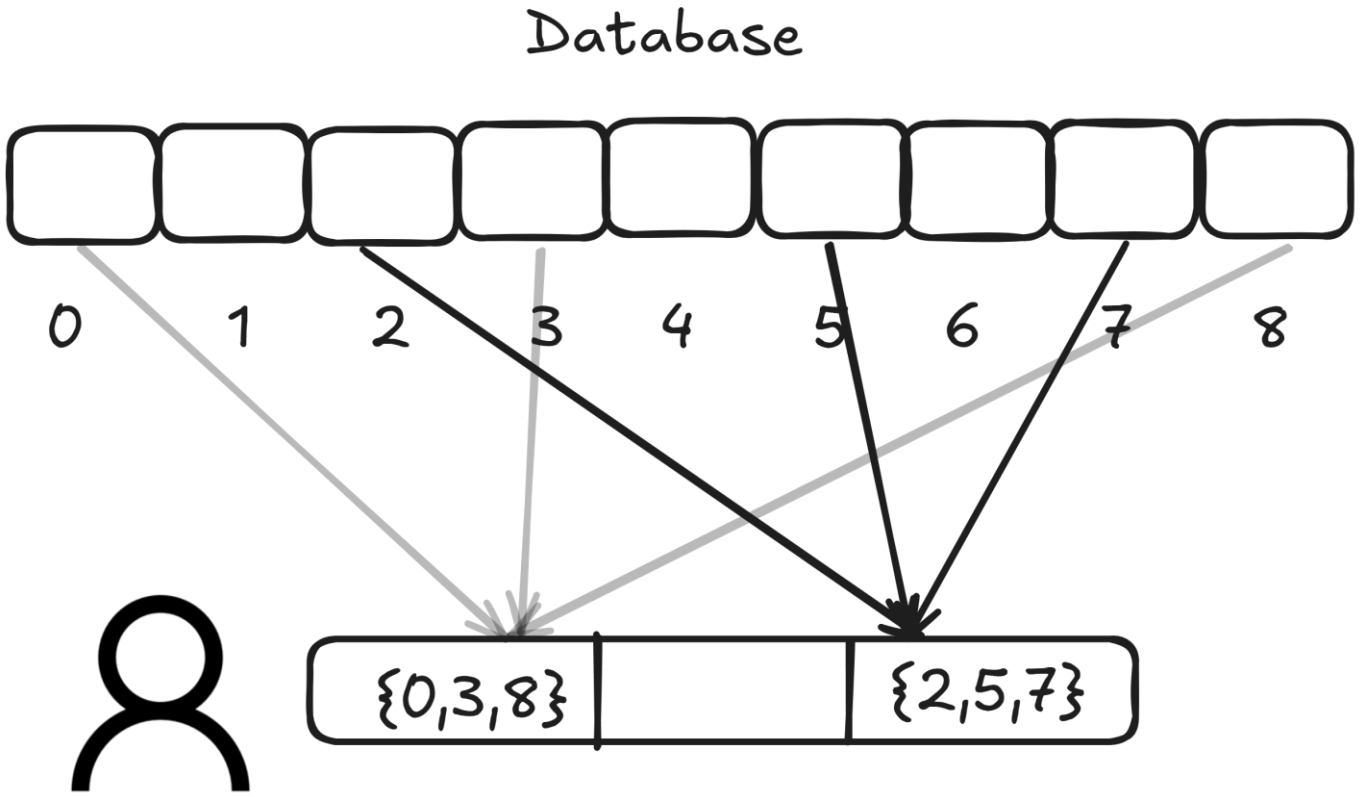
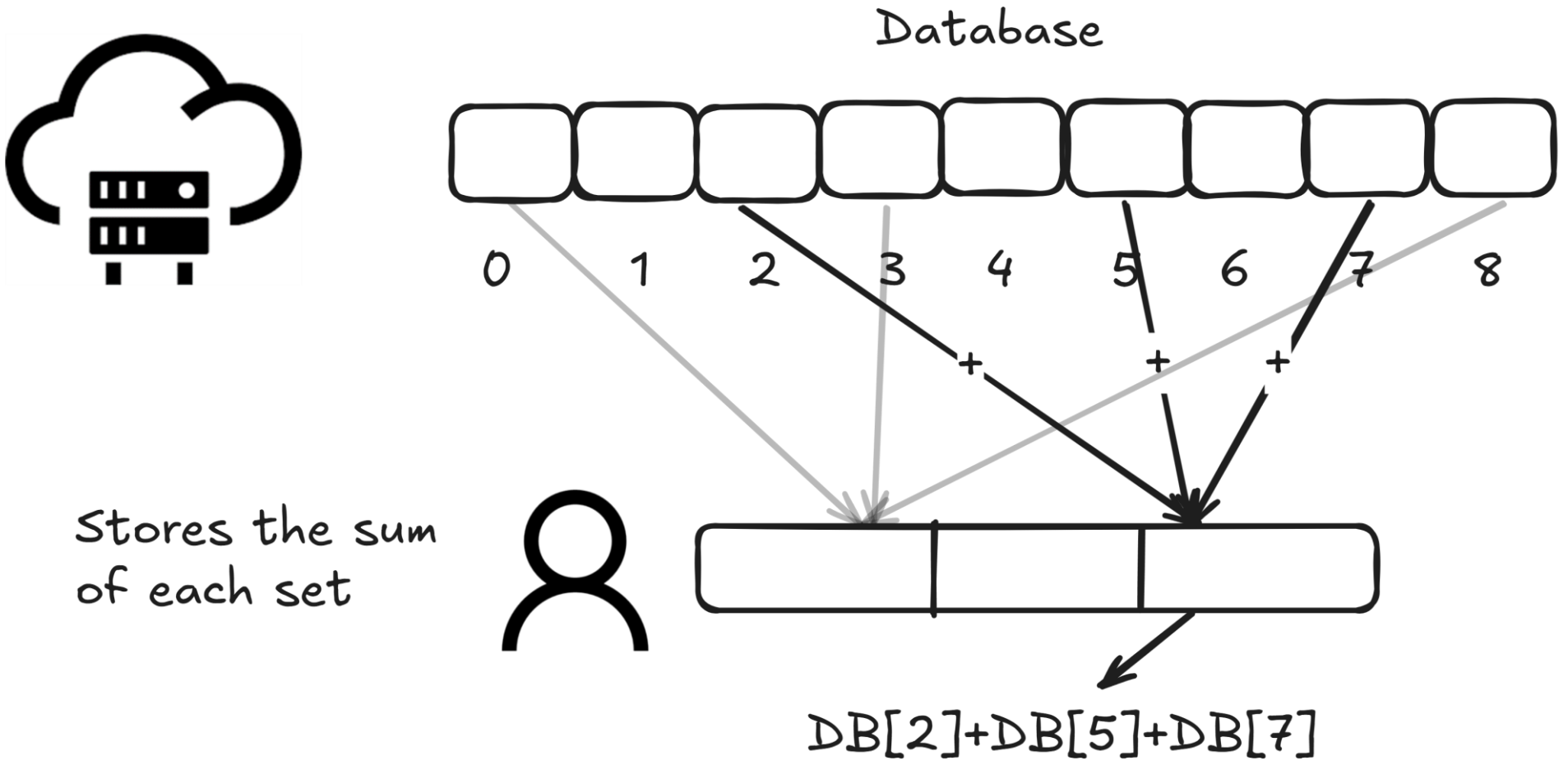


Local hint consists of sqrt(N) sets
Each set is represented by a PRF key
Each set has sqrt(N) random elements

Preprocessing



Query

what is DB[2]?

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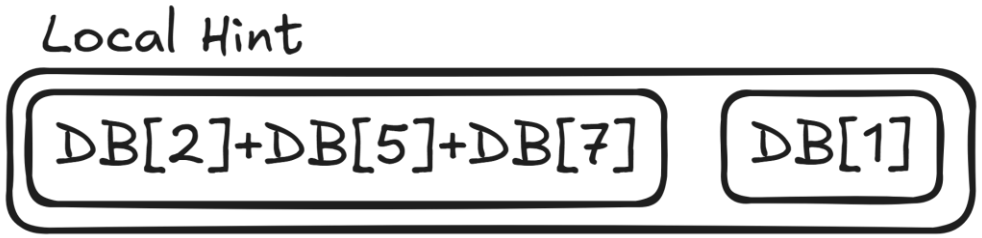

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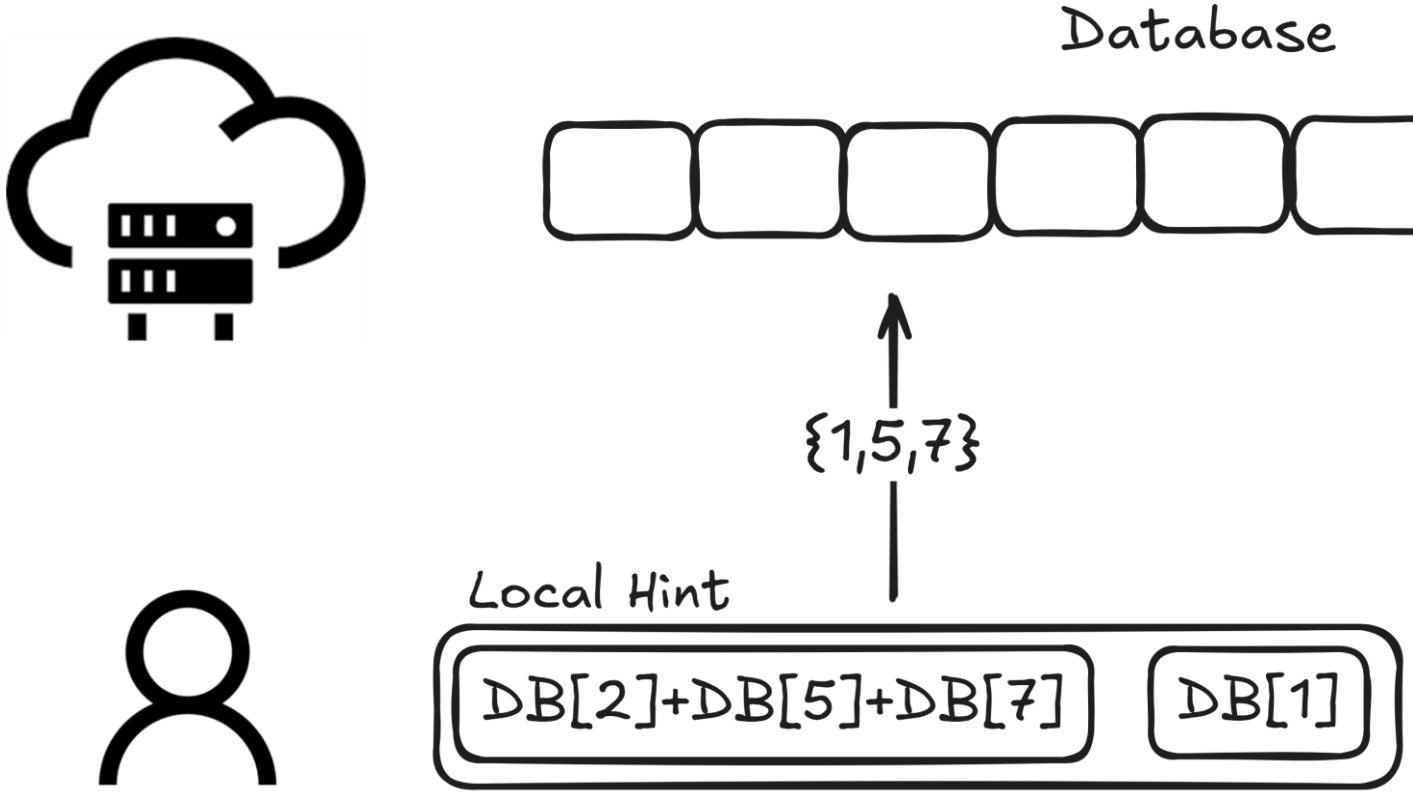

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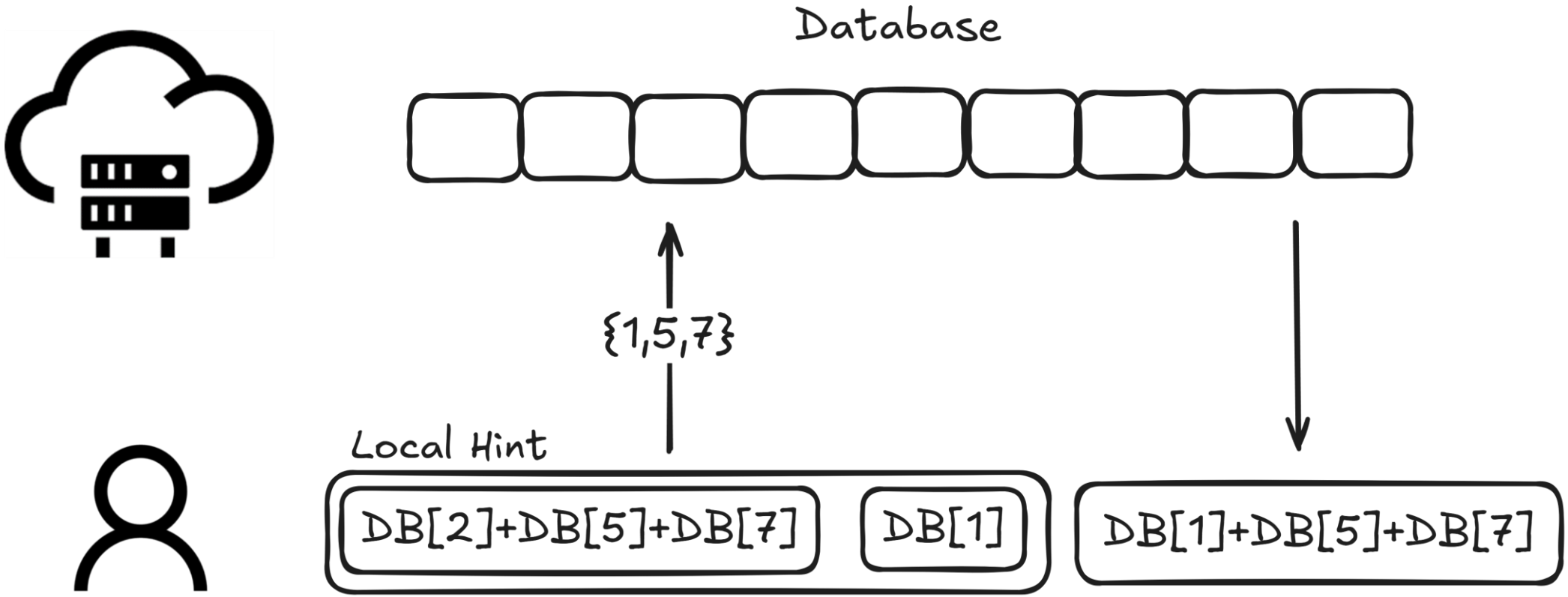

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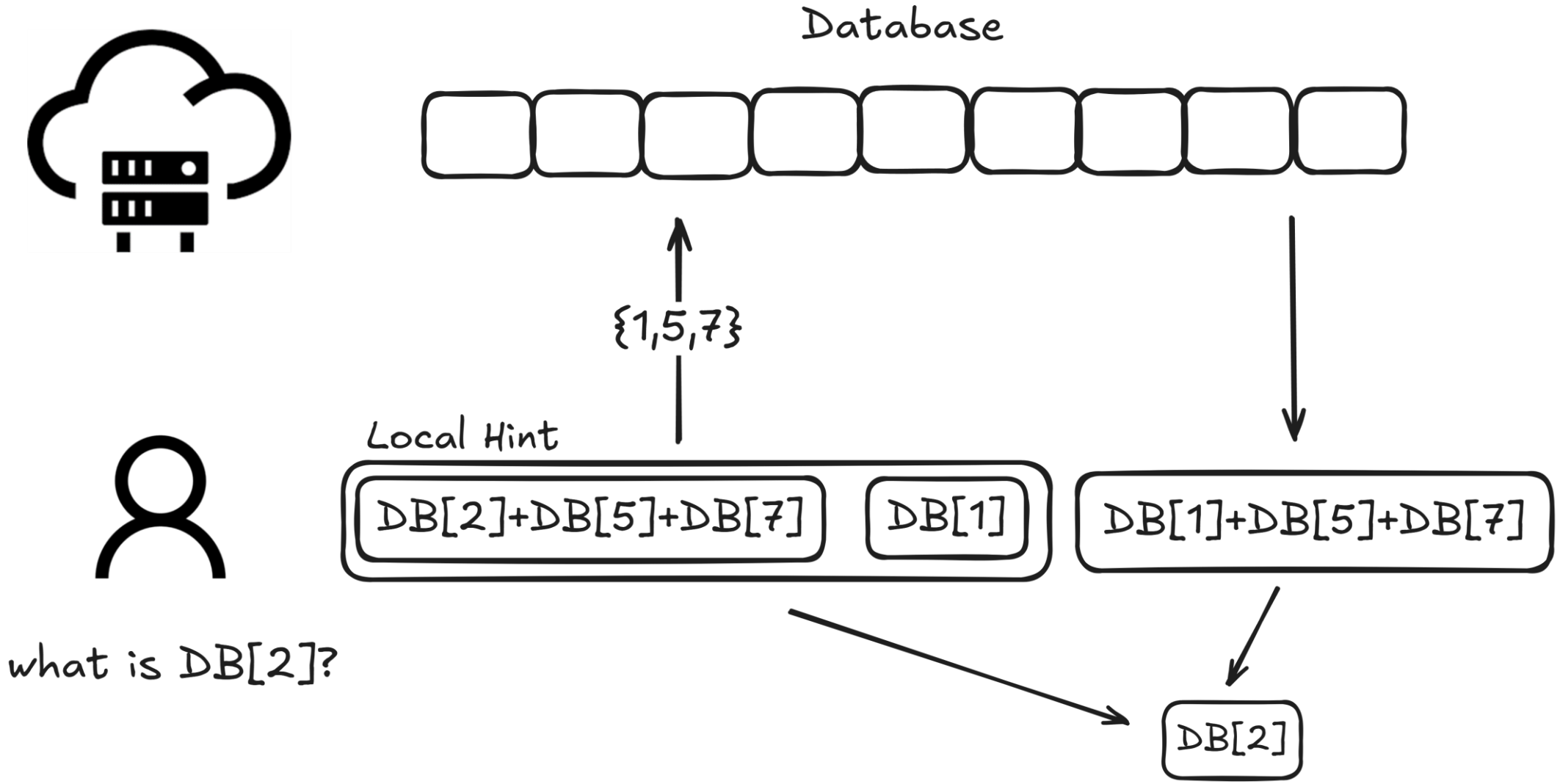
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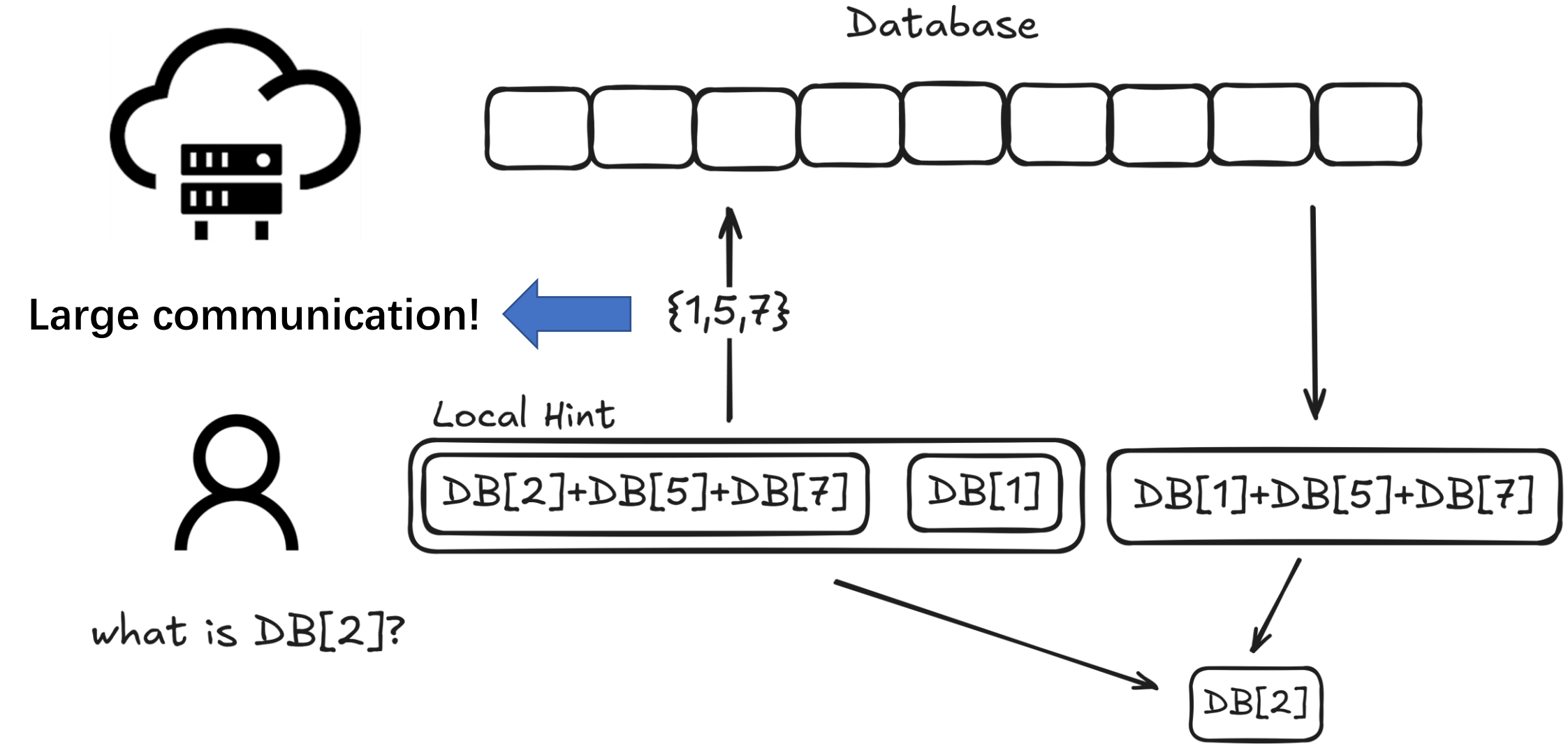
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- Can be constructed using classical GGM PRF

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- Applications: watermarking PRFs, MPC, PIR, ⋯

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- Can we construct PPPRF based on OWF, with efficient evaluation?

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- Expand a single key during Eval/Prog


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msk[0]
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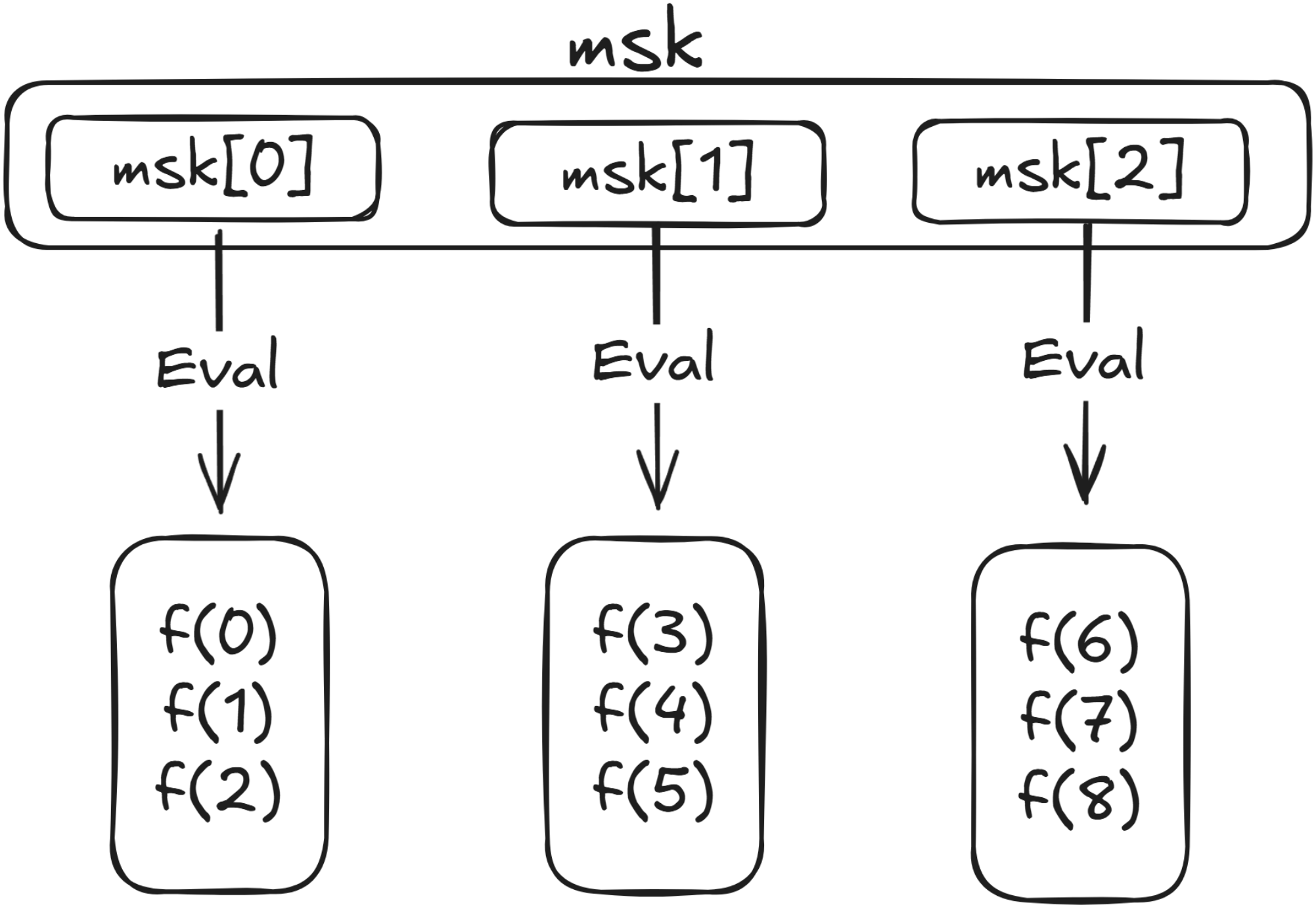
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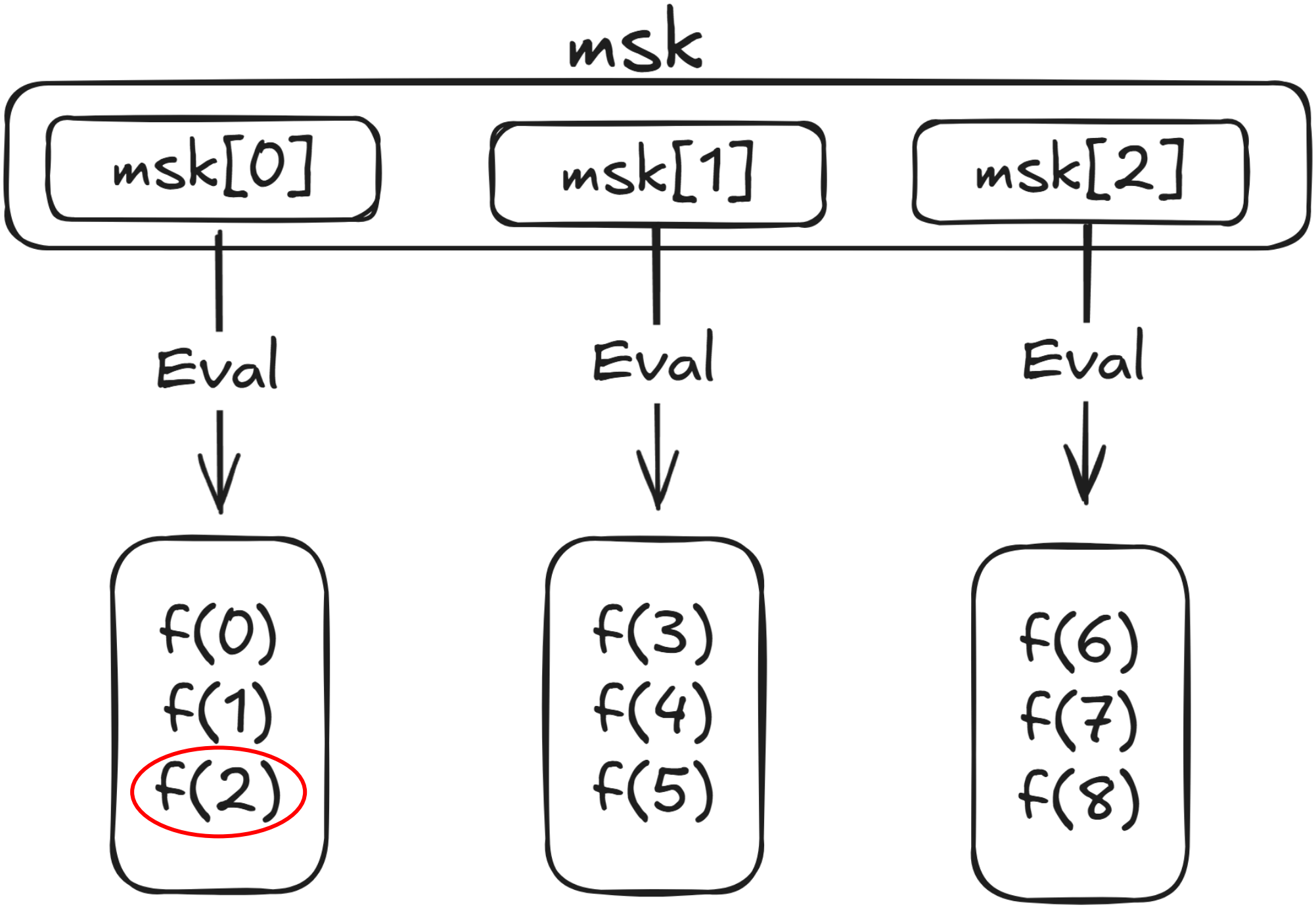
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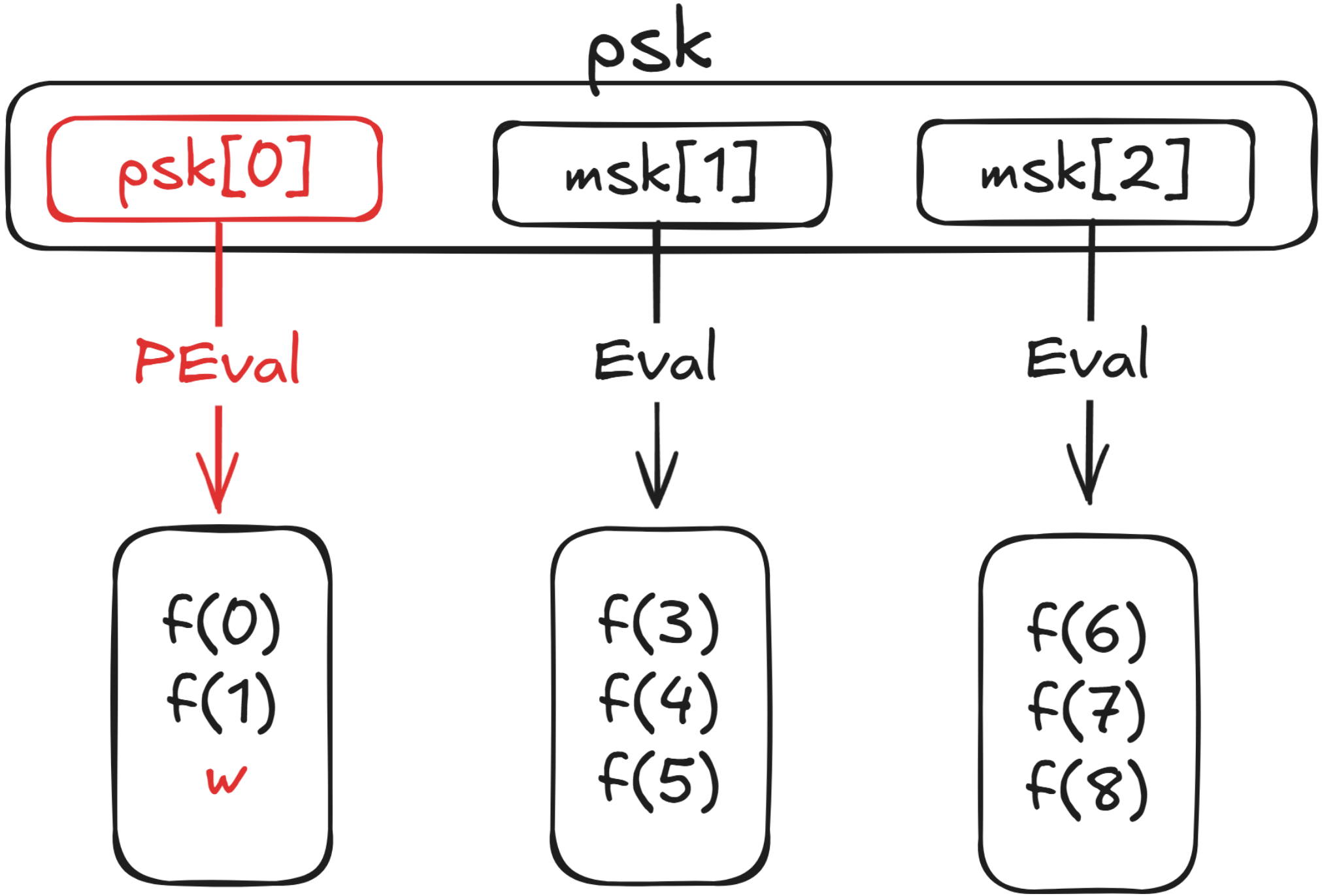
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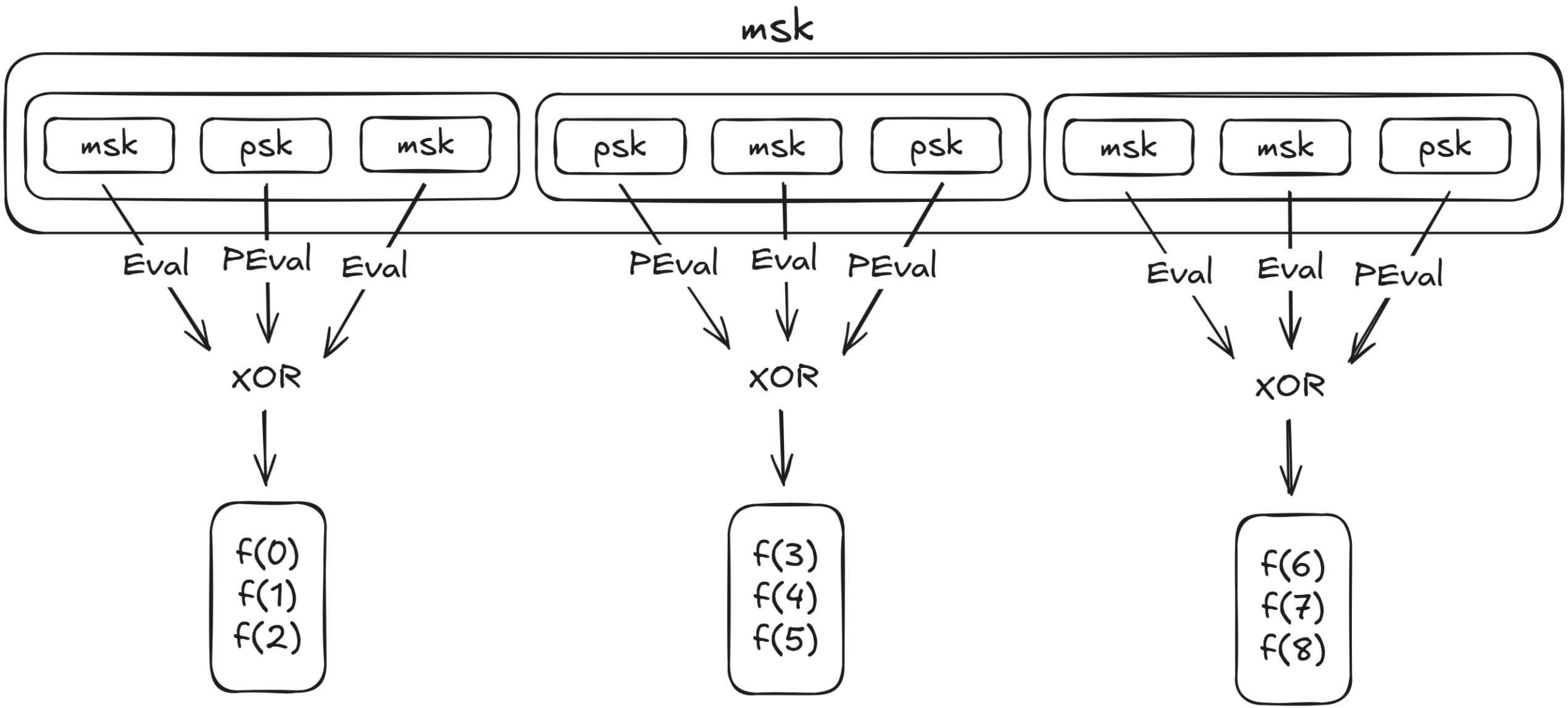
  - Can’t tell which block has one more programmed key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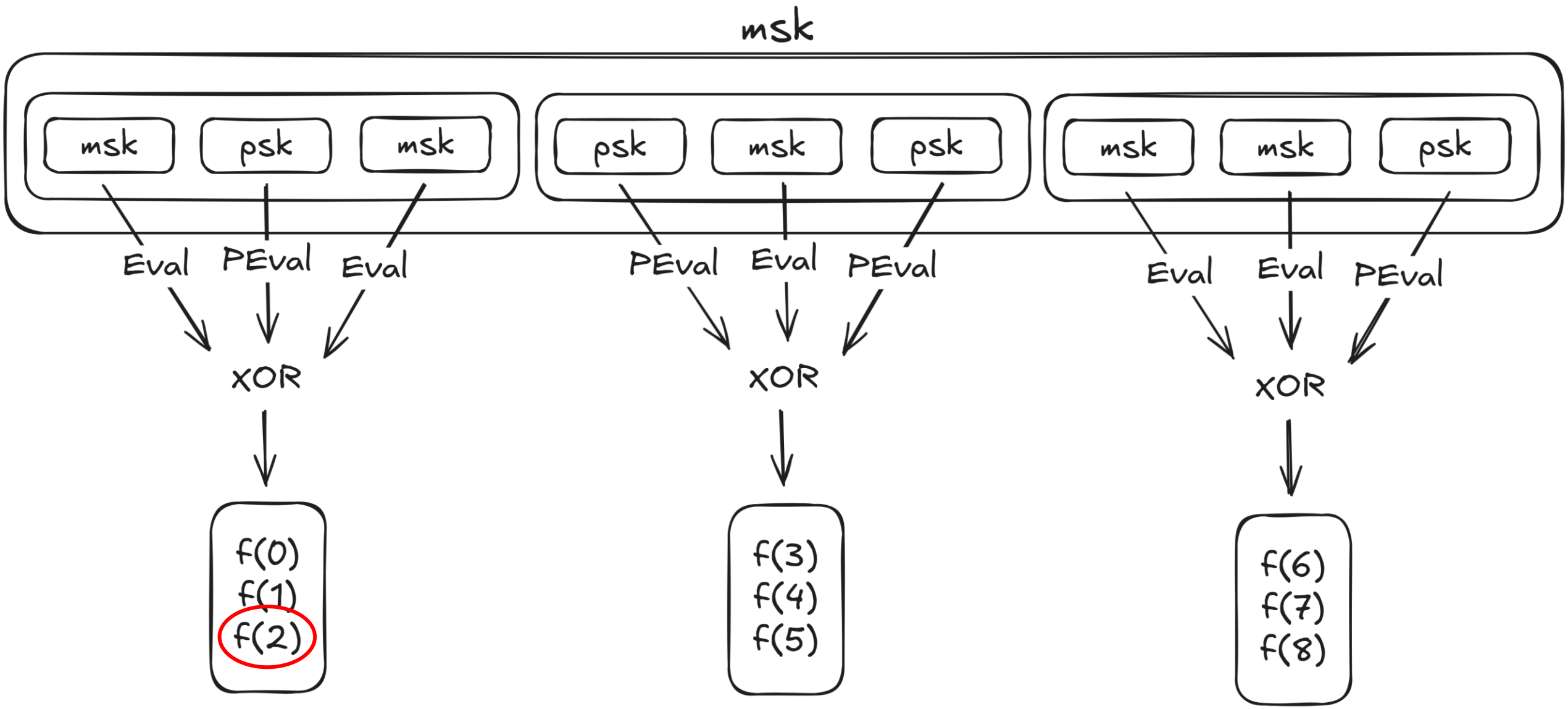
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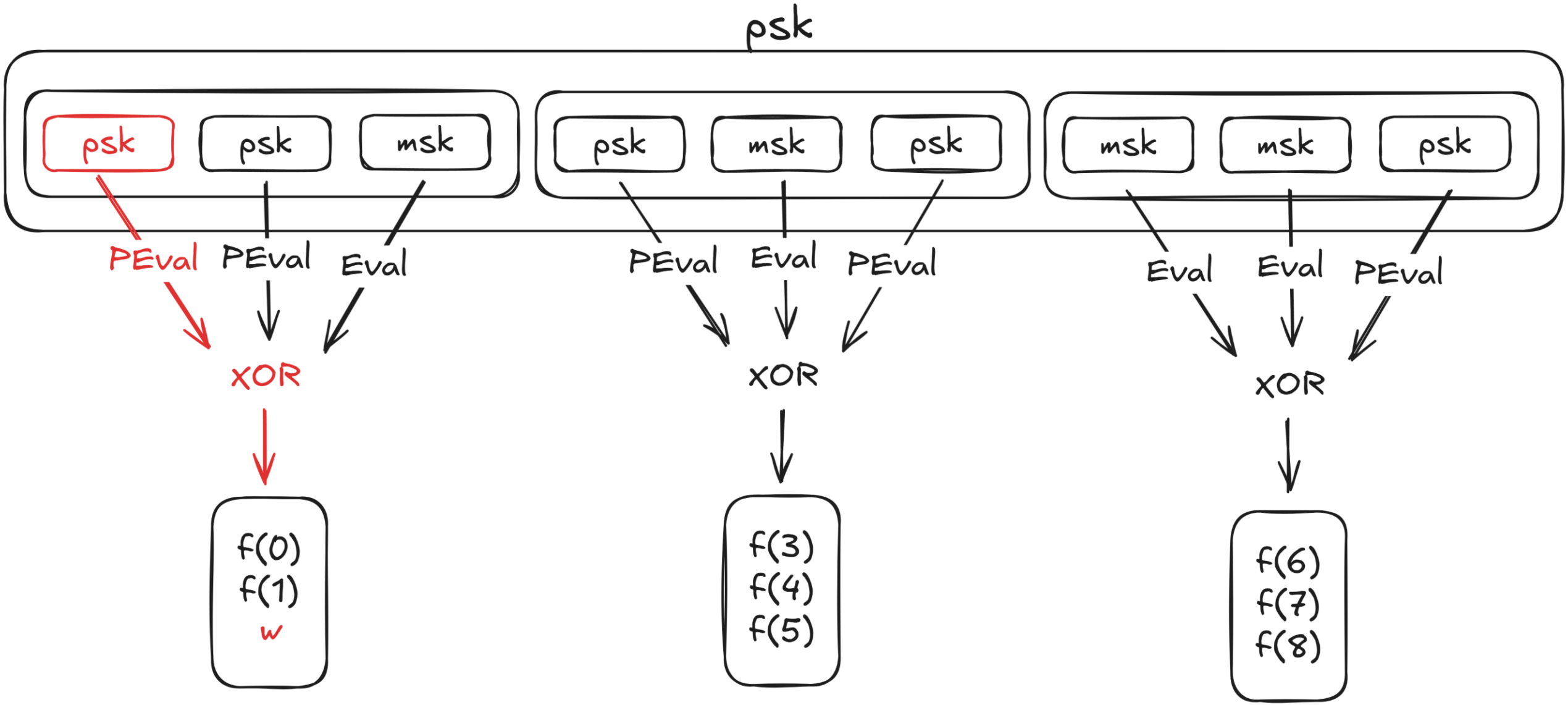
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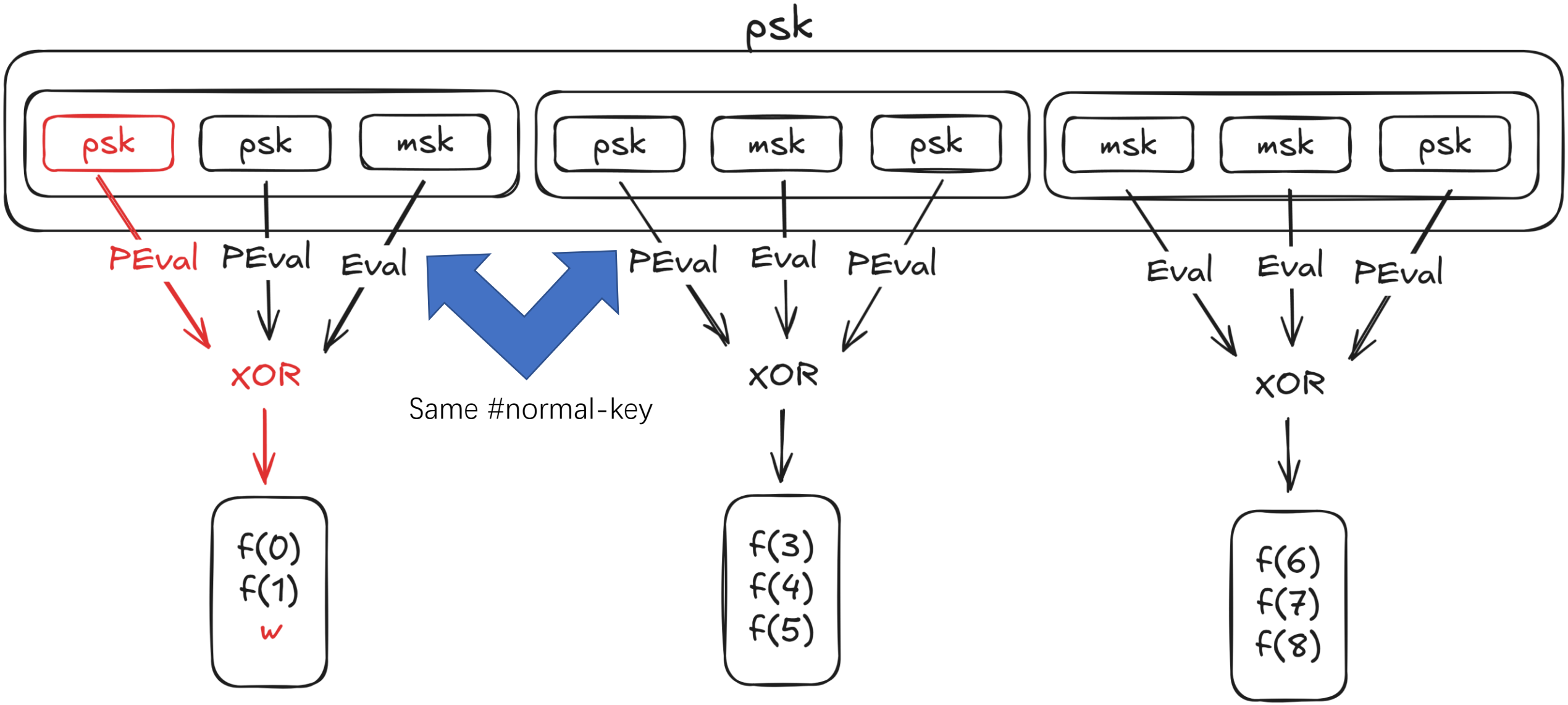
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- It achieves a notion of “weak” PPPRF:
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  - Relaxation of statistical distance
  - Intuitively, psk_y leaks y w.p. at most $1/\mathrm{poly}(\lambda)$

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- Use a PPPRF to generate the keys!

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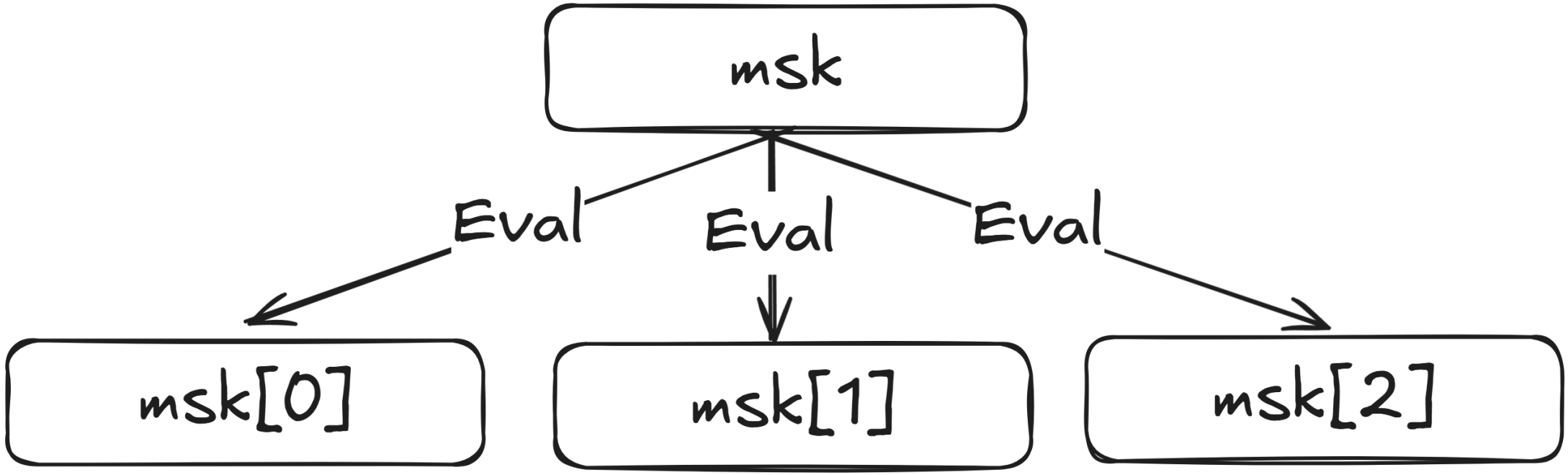
- Like a GGM PRF, but with larger fan-out


msk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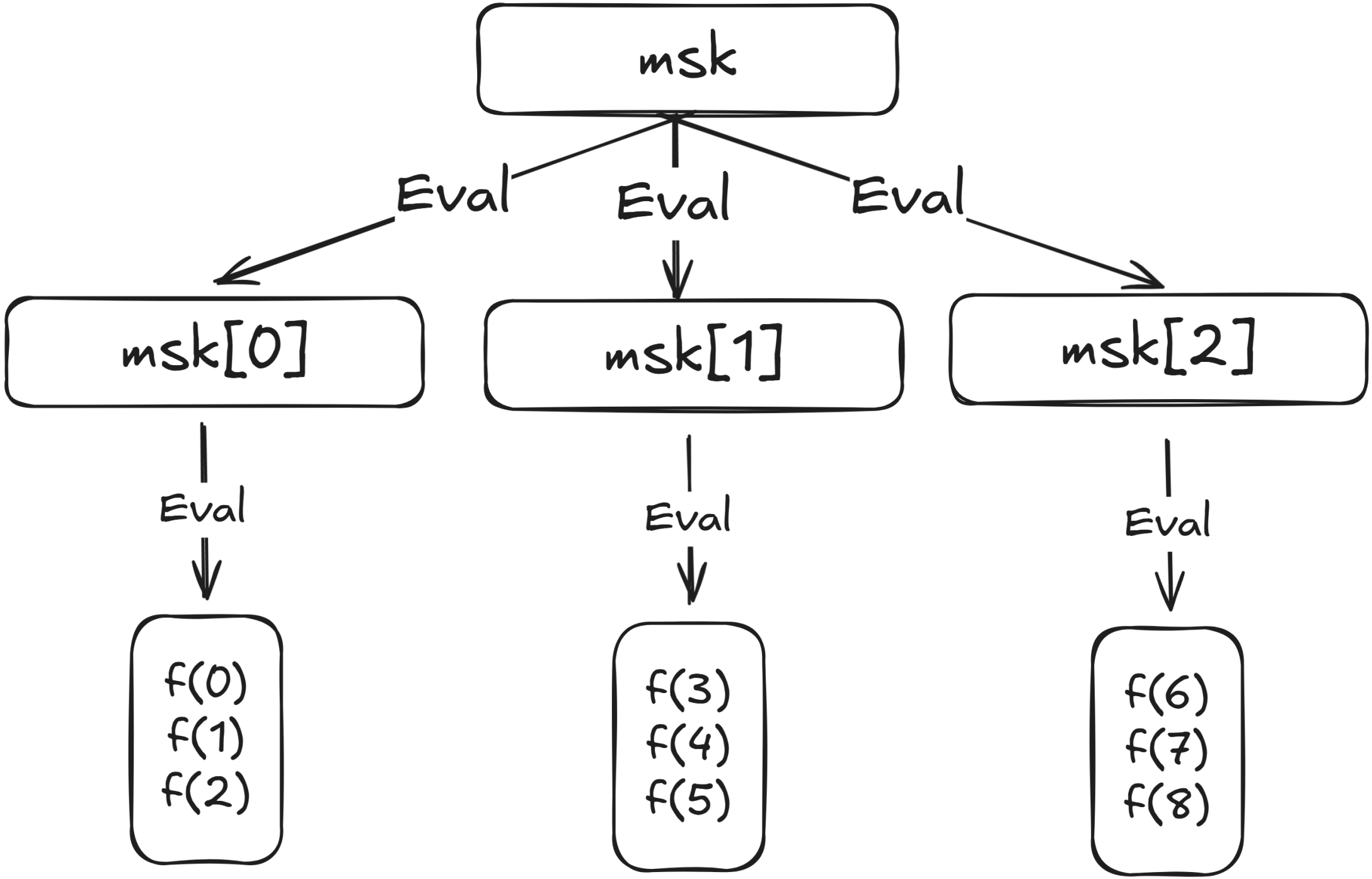
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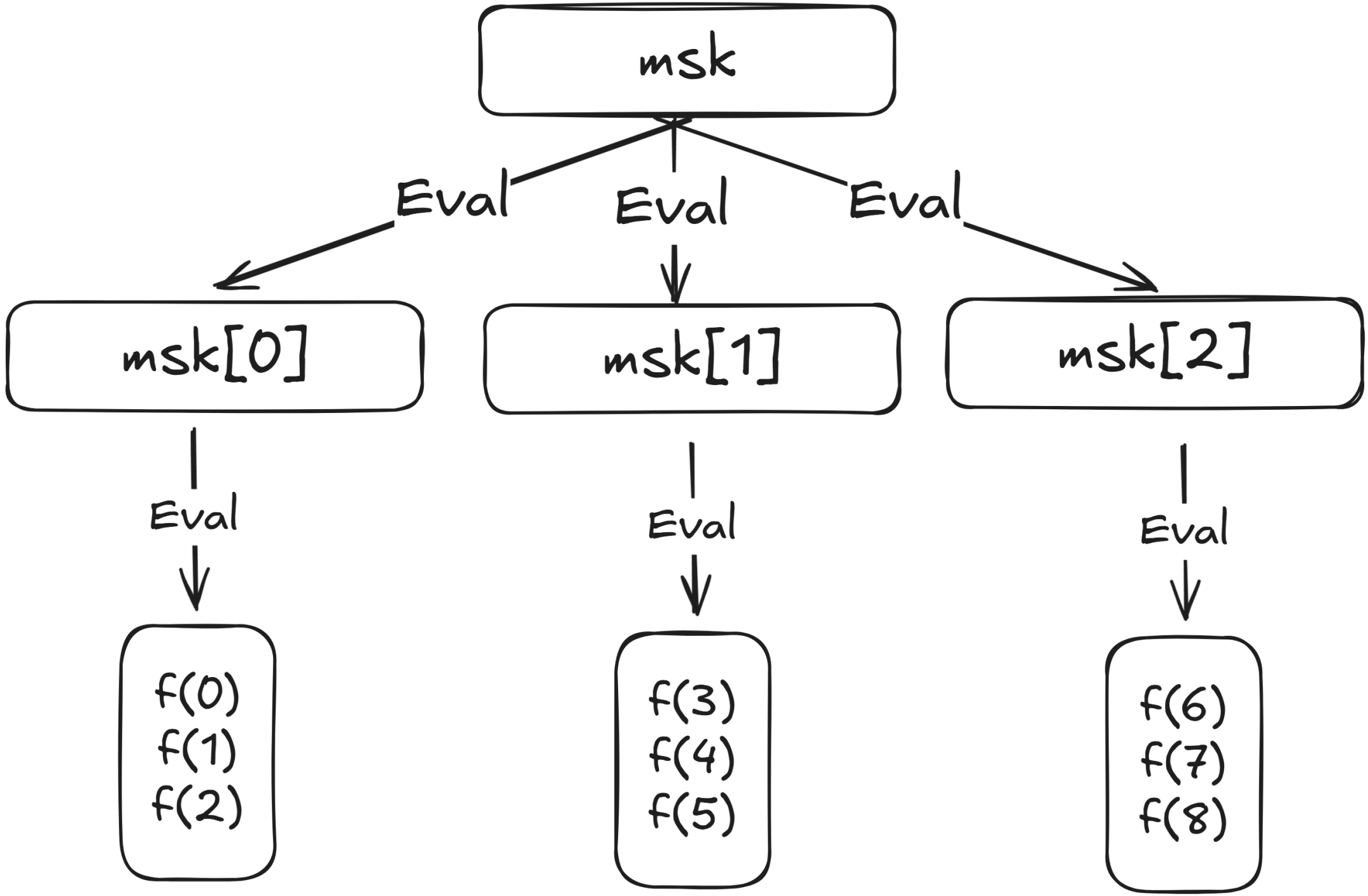
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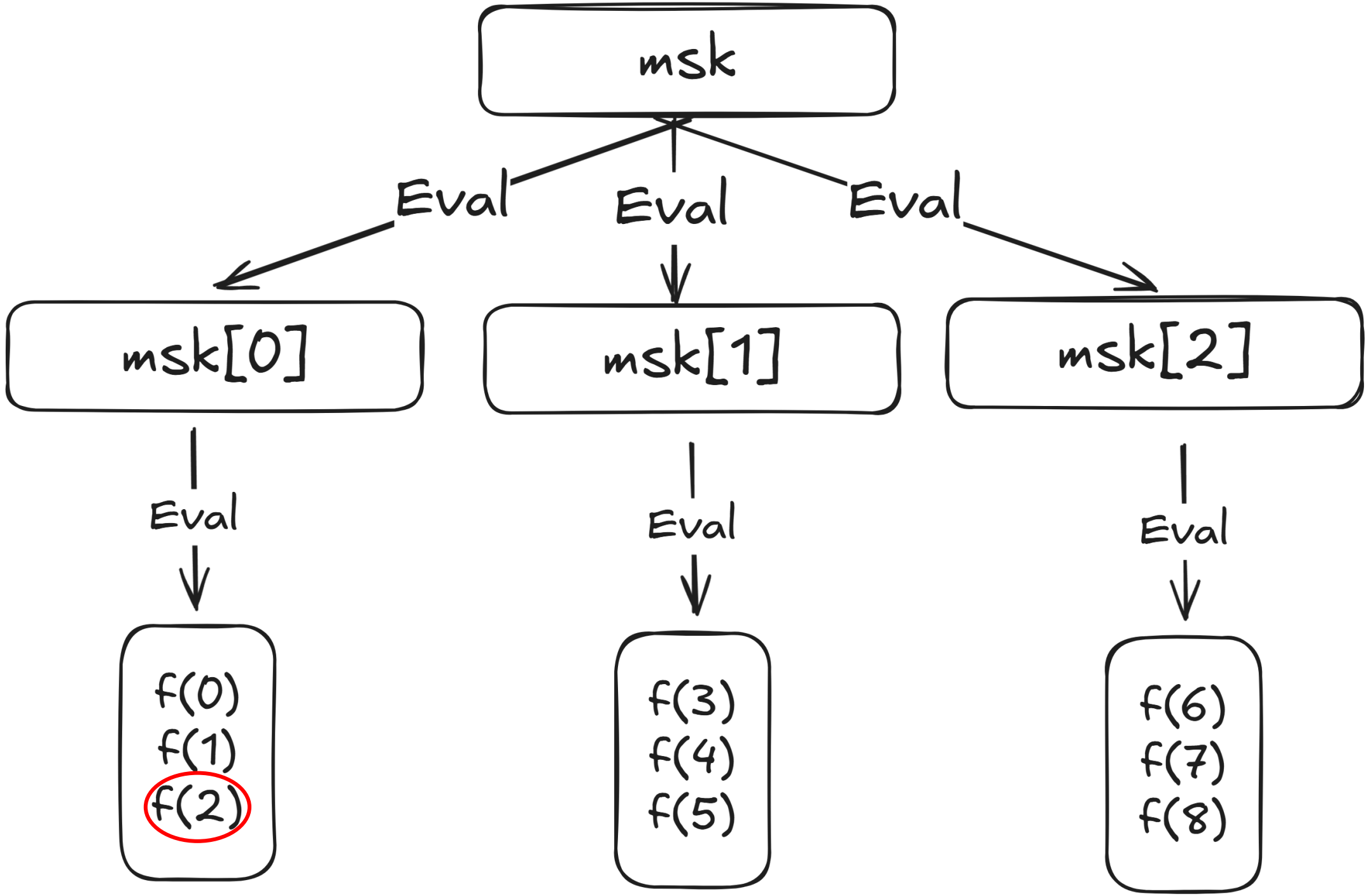
msk
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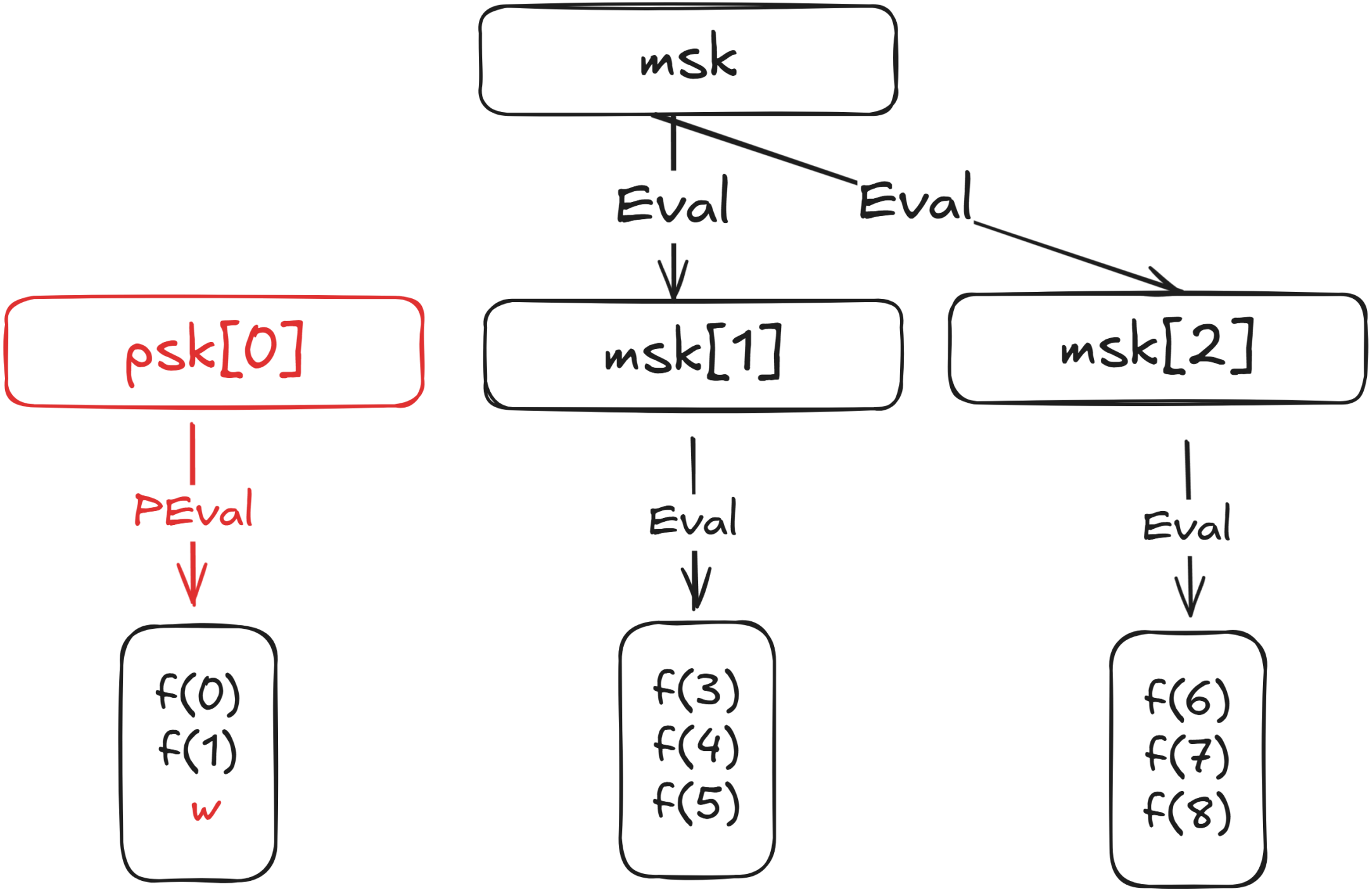
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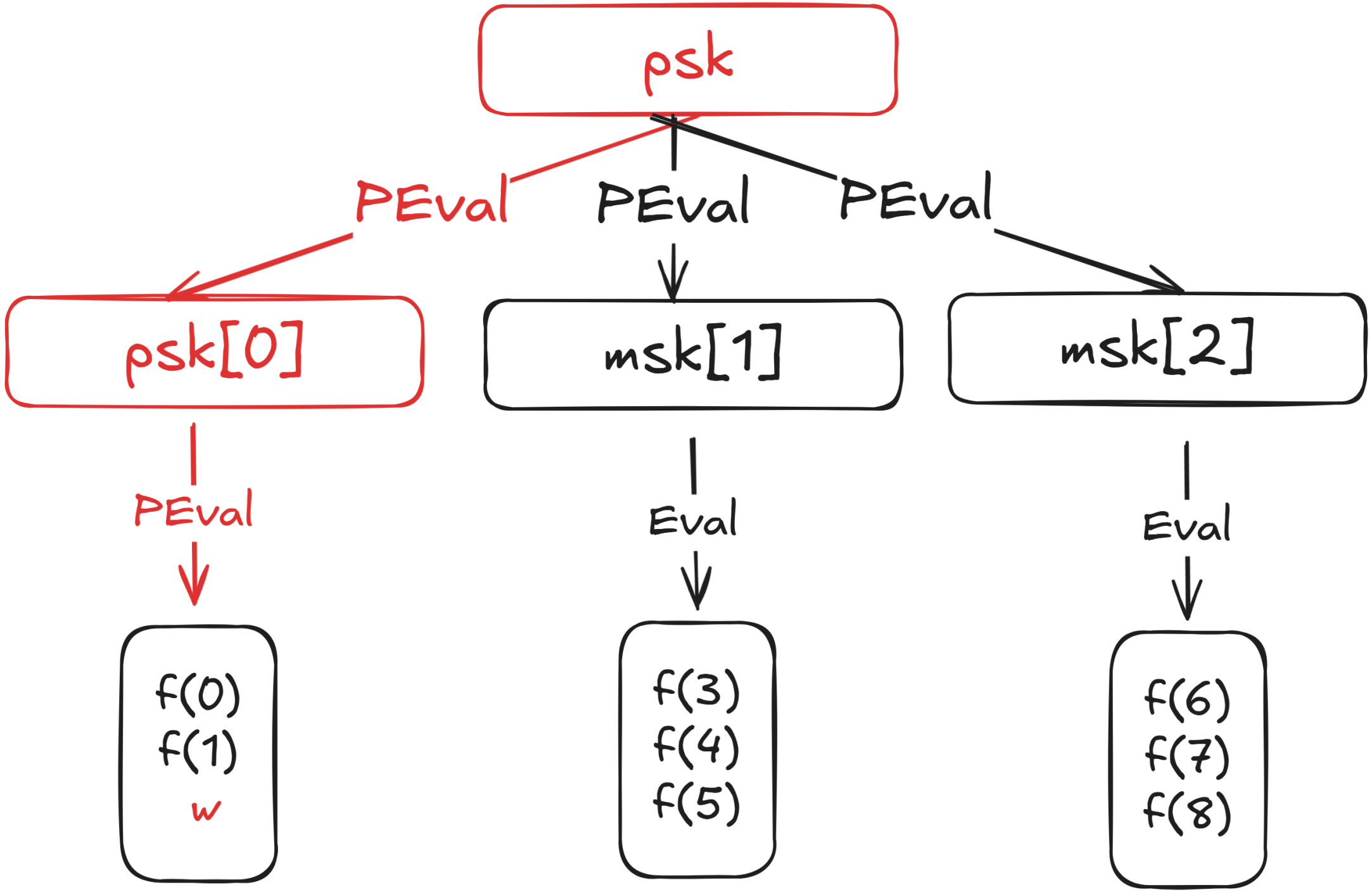
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Reducing the key size: Recursion!

- Key size: $O_\lambda(1)$
- Eval/Prog: $O_\lambda(n^{1/2})$

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- Can we do better?

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- Can we take it all the way to $\log n$ levels?
  - Sadly, no, because key size blows up exponentially

Summarize

- Assumption: OWF
- For arbitrary constant $0 < \epsilon < 1$:
  - Key size: $\lambda^{O(1/\epsilon)}$
  - Eval/Prog: $O_\lambda(n^\epsilon)$
- Security: $1/\mathrm{poly}(\lambda)$

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- Apply security amplification to PIR
  - See our paper for more details

Preprocessing PIR in Minicrypt

- Assuming $O_\lambda(n^{1/2})$ storage,

	Online Comm.	Offline comm.	Computation
Prev. best [ZPSZ24]	$O_\lambda(n^{1/4})$	$O_\lambda(n^{1/2})$	$O_\lambda(n^{1/2})$
Ours	$O_\lambda(1)$	$O_\lambda(n^{1/2+\epsilon})$	$O_\lambda(n^{1/2+\epsilon})$

- where $0 < \epsilon < 1$ is an arbitrary constant.

Future directions

- Concretely efficient PPPRF, based on LWE?
- More applications of security amplification?
- More primitives based on OWF?

- Read our paper:
  - ia.cr/2025/300

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Applying for PhD this year ☺